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CP violation in charged Higgs boson decays into tau and neutrino

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Abstract: We calculate the CP-violating rate asymmetry of $H^{\pm}$ decays into tau and neutrino at one loop in the MSSM with complex parameters. We find that the asymmetry is typically of the order of 10^{-3}, depending mainly on the phases of the trilinear coupling A_{τ} and the gaugino mass M_1.

Keywords: Supersymmetry Phenomenology, Supersymmetric Standard Model, Higgs Physics, CP violation.

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Contents

1. Introduction

The CP violation observed in the kaon system and in B meson decays appears to be consistent with the Standard Model (SM). However, the baryon asymmetry in the universe requires a new source of CP violation beyond the CKM phase of the SM. Indeed, most extensions of the SM provide additional sources of CP violation. Searching for new effects of CP violation has become one of the most interesting ways to test physics beyond the SM [1].

Within the Minimal Supersymmetric Standard Model (MSSM) with complex parameters, the new sources of CP violation are the phase of the higgsino mass parameter μ, two phases of the gaugino masses $M_i, i = 1, 2, 3$, and the phases of the trilinear couplings A_f. Especially the latter ones are practically unconstrained. Recently, we pointed out [2] that CP violation in the MSSM may lead to a difference in the partial decay widths of H^+ and H^-. More precisely, we showed that large phases of A_t, A_b and/or M_3 can give a CP-violating asymmetry $\delta_{tb}^{CP} = \left[\Gamma\left(H^+ \to t\bar{b}\right) - \Gamma\left(H^- \to \bar{t}b\right)\right] / \left[\Gamma\left(H^+ \to t\bar{b}\right) + \Gamma\left(H^- \to \bar{t}b\right)\right]$ of 10 – 15% for $m_{H^\pm} > m_{\tilde{t}} + m_{\tilde{b}}$.

In this article, we consider the lepton decay channels of the charged Higgs bosons, $H^+ \to \tau^+\nu_\tau$ and $H^- \to \tau^-\bar{\nu}_\tau$. In particular, we calculate the CP-violating asymmetry

$$\delta^{CP}_{\tau\nu} = \frac{\Gamma\left(H^+ \to \tau^+\nu_\tau\right) - \Gamma\left(H^- \to \tau^-\bar{\nu}_\tau\right)}{\Gamma\left(H^+ \to \tau^+\nu_\tau\right) + \Gamma\left(H^- \to \tau^-\bar{\nu}_\tau\right)}\,, \tag{1.1}$$

at the one-loop level in the MSSM with explicit CP violation and discuss its parameter dependence. The decay into $\tau\nu$ may be important for relatively low masses of $H^\pm$ and large $\tan\beta$ as its branching ratio increases with increasing $\tan\beta$. For example, for $m_{H^+} = 250\,\mathrm{GeV}$, we have $Br(H^+ \to t\bar{b}) = 0.90$ and $Br(H^+ \to \tau^+\nu_\tau) = 0.06$ for $\tan\beta = 5$, and $Br(H^+ \to t\bar{b}) = 0.64$ and $Br(H^+ \to \tau^+\nu_\tau) = 0.36$ for $\tan\beta = 30$. The asymmetry $\delta^{CP}_{\tau\nu}$ is sensitive to the phases of the trilinear coupling A_τ and of the gaugino mass M_1. Although one expects $\delta^{CP}_{\tau\nu}$ to be smaller than δ^{CP}_{tb} due to the missing gluino exchange, it is an interesting quantity in the case $m_{\tilde{\tau}} + m_{\tilde{\nu}_\tau} < m_{H^\pm} < m_{\tilde{t}} + m_{\tilde{b}}$.

The article is organized as follows: in section 2 we derive the basic formulae for the $H^\pm \to \tau\nu$ decay widths and define $\delta^{CP}_{\tau\nu}$ in terms of CP-violating form factors δY^{CP}_τ. In section 3, we perform a numerical analysis. In section 4, we summarize our results and comment on the measurability of $\delta^{CP}_{\tau\nu}$. Appendices A and B contain the explicit formulae for the form factors, masses and couplings.

2. Decay widths at tree level and one loop

At lowest order, the widths of the $H^\pm \to \tau^\pm \overset{(-)}{\nu}_\tau$ decays are given by

$$\Gamma^{LO}\left(H^\pm \to \tau\nu\right) = \frac{(m^2_{H^\pm} - m^2_\tau)^2\, y^2_\tau}{16\pi m^3_{H^\pm}}\,, \tag{2.1}$$

where $y_\tau = h_\tau \sin\beta$, h_τ being the tau Yukawa coupling, $h_\tau = g m_\tau / \left(\sqrt{2}\, m_W \cos\beta\right)$. At tree level, the amplitude is real and the decay widths are always equal, $\Gamma^{LO}(H^+ \to \tau^+\nu_\tau) = \Gamma^{LO}(H^- \to \tau^-\bar{\nu}_\tau)$. However, once loop corrections with complex couplings are included, we have $y_\tau \to Y^\pm_\tau = y_\tau + \delta Y^\pm_\tau$ and thus a difference in the decay widths appears. At next-to-leading order we get:

$$\Gamma^{NLO}\left(H^\pm \to \tau\nu\right) = \frac{(m^2_{H^\pm} - m^2_\tau)^2\, y^2_\tau}{16\pi m^3_{H^\pm}} \left(1 + \frac{2\ \mathrm{Re}\ \delta Y^\pm_\tau}{y_\tau}\right). \tag{2.2}$$

Here δY^+_τ stands for the decay of H^+ and δY^-_τ for the decay of H^-. The radiative corrections $\delta Y^\pm_\tau$ have, in general, both CP-invariant and CP-violating contributions:

$$\delta Y^\pm_\tau = \delta Y^{inv}_\tau \pm \frac{1}{2}\,\delta Y^{CP}_\tau\,. \tag{2.3}$$

Both the CP-invariant and the CP-violating contributions have real and imaginary parts. Using eqs. (2.2) and (2.3), we can write the CP-violating asymmetry $\delta^{CP}_{\tau\nu}$ of eq. (1.1) in the simple form:

$$\delta^{CP}_{\tau\nu} = \frac{\mathrm{Re}\ \delta Y^{CP}_\tau}{y_\tau\ +\ 2\ \mathrm{Re}\ \delta Y^{inv}_\tau} \simeq \frac{\mathrm{Re}\ \delta Y^{CP}_\tau}{y_\tau}\,. \tag{2.4}$$

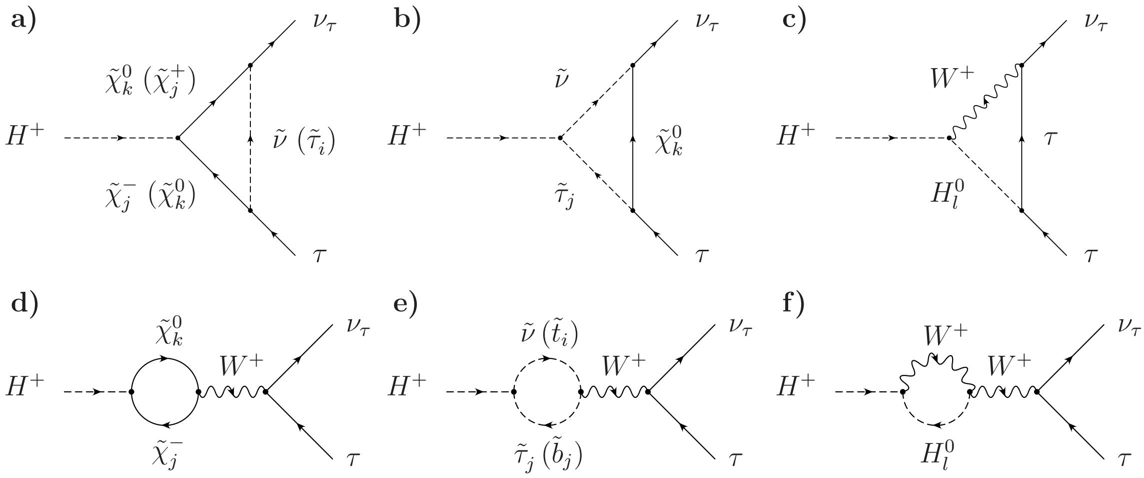

Figure 1: Diagrams contributing to CP violation in $H^+ \to \tau^+\nu_\tau$ in the MSSM with complex couplings ($j = 1, 2$; $k = 1, \ldots, 4$; $l = 1, 2, 3$).

In the MSSM, $\delta^{CP}_{\tau\nu}$ gets contributions from loop exchanges of charginos, neutralinos, sfermions, W bosons, and neutral Higgs bosons. The relevant Feynman diagrams are shown in figure 1. Note that the various diagrams contribute to $\delta^{CP}_{\tau\nu}$ only if they have absorptive parts. The form factors δY^{CP}_τ can be obtained from δY^{CP}_b in [2] by the replacements (s)bottom $\to$ (s)tau and (s)top $\to$ (s)neutrino. Since $m_\nu \sim h_\nu \sim 0$, many terms vanish and δY^{CP}_τ becomes much simpler than δY^{CP}_b. The explicit expressions for δY^{CP}_τ due to the various diagrams of figure 1, together with the masses and couplings of staus and sneutrinos, are given in the appendix A and B. All other necessary formulae can be found in [2].

3. Numerical results

The parameters relevant to our study are the gaugino mass parameters M_1 and M_2; the higgsino mass parameter μ; the soft-breaking parameters of the tau-sleptons $M_{\tilde{L}}$, $M_{\tilde{E}}$ and A_τ; the charged Higgs boson mass m_{H^+}, and $\tan\beta = v_2/v_1$. We also need the parameters of the stop-sbottom sector, $M_{\tilde{Q},\tilde{U},\tilde{D}}$, and $A_{t,b}$: on one hand for the $\tilde{t}\tilde{b}$ self-energy diagram of figure 1e, on the other hand for the radiative corrections to the neutral Higgs sector.

Quite generally, the gaugino and higgsino mass parameters M_1, M_2, μ and the trilinear couplings of the third generation $A_{t,b,\tau}$ can have physical phases that may lead to sizable CP-violating effects. In order not to vary too many parameters, we fix part of the parameter space by

$$M_2 = 200\,\mathrm{GeV}\,, \quad \mu = 300\,\mathrm{GeV}\,, \quad M_{\tilde{E}} = M_{\tilde{L}} - 5\,\mathrm{GeV}\,, \quad |A_\tau| = 400\,\mathrm{GeV}\,, \tag{3.1}$$

$$M_{\tilde{Q}} = 500\,\mathrm{GeV}\,, \quad M_{\tilde{U}} = 450\,\mathrm{GeV}\,, \quad M_{\tilde{D}} = 550\,\mathrm{GeV}\,, \quad A_t = A_b = -500\,\mathrm{GeV}\,. \tag{3.2}$$

For M_1, we assume $|M_1| = (5/3)\tan\theta_W\,|M_2|$, keeping ϕ_1, the phase of M_1, as a physical phase. The phase of M_2 can be rotated away. Since according to the measurements of the electron and neutron electric dipole moments we have $\phi_\mu < \mathcal{O}(10^{-2})$ [3] for SUSY masses

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$\tan\beta$	$M_{\tilde{L}}$	$m_{\tilde{\nu}}$	$m_{\tilde{\tau}_2}$	$\theta_{\tilde{\tau}}$
5	138	123	150	56^o
10	147	132	166	50^o
30	180	168	221	47^o

Table 1: Parameters used in the analysis, masses in [GeV], $m_{\tilde{\tau}_1} = 135\,\text{GeV}$.

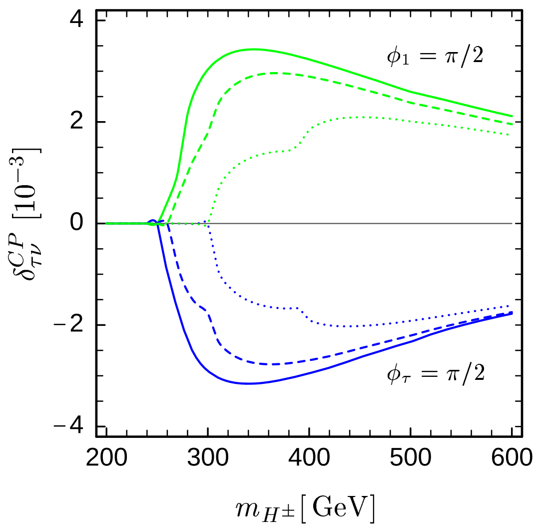

Figure 2: δ^{CP} as a function of m_{H^+} for $\phi_\tau = \pi/2$, $\phi_1 = 0$ ($\delta^{CP} < 0$), and for $\phi_1 = \pi/2$, $\phi_\tau = 0$ ($\delta^{CP} > 0$). The full, dashed, and dotted lines are for $\tan\beta = 5$, 10, and 30, respectively.

of the order of a few hundred GeV, we take $\phi_\mu = 0$. The remaining phases in our analysis are thus ϕ_t, ϕ_b and ϕ_τ (the phases of A_t, A_b and A_τ) and ϕ_1. These phases also induce, at one-loop level, a mixing of the CP-even and CP-odd neutral Higgs boson states to form mass eigenstates H_i^0, i=1,2,3 [4]. We take this mixing into account using [5].

Figure 2 shows $\delta^{CP}_{\tau\nu}$ as a function of $m_{H^\pm}$ for the two cases $\phi_\tau = \pi/2$, $\phi_1 = 0$ and $\phi_\tau = 0$, $\phi_1 = \pi/2$ (all other phases zero) and three values of $\tan\beta$: $\tan\beta = 5$, 10, and 30. $M_{\tilde{L}}$ is chosen such that the lighter stau mass is $m_{\tilde{\tau}_1} = 135\,\text{GeV}$. The corresponding values for $m_{\tilde{\nu}}$, $m_{\tilde{\tau}_2}$ and $\theta_{\tilde{\tau}}$ are listed in table 1.

The dominant source of CP violation in $H^\pm \to \tau^\pm \overset{(-)}{\nu}_\tau$ decays is the sneutrino-stau-neutralino loop of figure 1b: For $m_{H^\pm} < m_{\tilde{\tau}_1} + m_{\tilde{\nu}}$, $\delta^{CP}_{\tau\nu}$ is negligibly small, while it sharply rises once the $H^\pm \to \tilde{\tau}_1 \tilde{\nu}_\tau$ channel is open. In figure 2, $|\delta^{CP}_{\tau\nu}|$ goes up to $\sim 3.5 \times 10^{-3}$; in our analysis, we have not found values larger than 0.5% (though we do not exclude them for some extreme values of MSSM parameters). Here note that we have taken a rather large value for $|A_\tau|$ compared to $M_{\tilde{L}}$. For smaller $|A_\tau|$, $\delta^{CP}_{\tau\nu}$ typically decreases. $\delta^{CP}_{\tau\nu}$ also decreases with increasing $\tan\beta$.

It is interesting to note that maximal ϕ_τ and maximal ϕ_1 lead to very similar values of $\delta^{CP}_{\tau\nu}$ but with opposite signs. However, if both phases are maximal, i.e. $\phi_t \sim \phi_1 \sim \pi/2$ or $3\pi/2$, they compensate each other and $\delta^{CP}_{\tau\nu}$ practically vanishes. In figure 3, $\delta^{CP}_{\tau\nu}$ is shown as a function of ϕ_τ for $m_{H^\pm} = 350\,\text{GeV}$, $\tan\beta = 5$, and various values of ϕ_1. One sees that ϕ_τ and ϕ_1 are of equal importance for $\delta^{CP}_{\tau\nu}$.

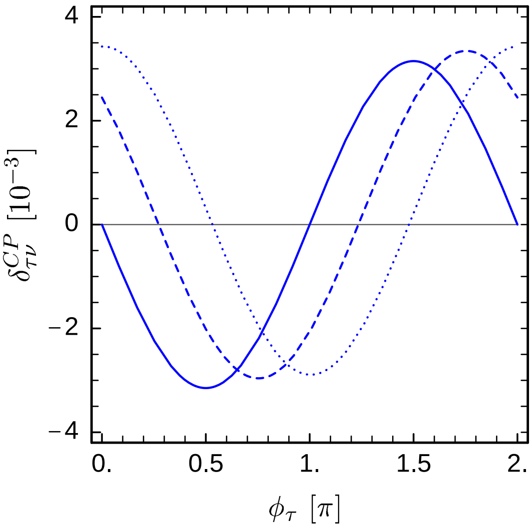

Figure 3: δ^{CP} as a function of ϕ_τ for $m_{H^\pm} = 350\,\text{GeV}$ and $\tan\beta = 5$. The full, dashed, and dotted lines are for $\phi_1 = 0$, $\pi/4$, and $\pi/2$, respectively.

We have also examined the dependence on $\phi_{t,b}$. Here notice that in the considered range of $m_{H^\pm}$, $200\ \text{GeV} < m_{H^\pm} < 600\ \text{GeV}$, the diagram with $\tilde{t}\,\tilde{b}$, figure 1e, does not contribute. Thus the parameters of the squark sector, eq. (3.2), enter only through radiative corrections to the neutral Higgs sector, see diagrams 1c and 1f, which turn out to be negligible in the case $m_{H^\pm} > m_{\tilde{\tau}_1} + m_{\tilde{\nu}}$. For completeness we note that also a non-zero ϕ_μ has only little influence on $\delta^{CP}_{\tau\nu}$.

4. Conclusions

We have calculated the one-loop contributions to the decays $H^\pm \to \tau^\pm \overset{(-)}{\nu}_\tau$ within the MSSM with complex parameters. They lead to a CP-violating asymmetry $\delta^{CP}_{\tau\nu}$, eq. (1.1), different from zero. The relevant phases in our analysis are those of the trilinear coupling A_τ and the gaugino mass M_1. For $m_{H^+} > m_{\tilde{\tau}_1} + m_{\tilde{\nu}}$, the asymmetry is typically of order 10^{-3}, the dominant source being the sneutrino-stau-neutralino loop.

Some comments are in order on the feasibility of measuring such an asymmetry. As already mentioned, the branching ratio for $H^\pm \to \tau^\pm \overset{(-)}{\nu}_\tau$ is sizeable only for $\tan\beta > 10$. At the LHC, the dominant production channel for H^+ is $gb \to H^+ t$. One expects [6] 1560 events for a Higgs mass of $m_{H^\pm} = 400\,\text{GeV}$ and $\tan\beta = 50$ with a ratio signal over background $S/\sqrt{B} = 19.8$ with an integrated luminosity of 100 fb^{-1}. (Here several cuts were already applied and b tagging assumed.) With a branching ratio of 22% one then has 343 events of $H^+ \to \tau^+\nu_\tau$.

At a linear e^+e^- collider at $\sqrt{s} = 1\,\text{TeV}$, the cross section of $e^+e^- \to H^+H^-$ for $m_{H^\pm} = 400\ \text{GeV}$ is 6.5 fb. Assuming a luminosity of $\mathcal{L} = 500\ \text{fb}^{-1}$ and again a branching ratio of 22% for $H^+ \to \tau^+\nu_\tau$ at $\tan\beta = 50$, one gets 715 events. At CLIC with $\sqrt{s} = 3\,\text{TeV}$ the cross section for $e^+e^- \to H^+H^-$ for $m_{H^\pm} = 400\,\text{GeV}$ is 3 fb. With $\mathcal{L} = 800\ \text{fb}^{-1}$ one gets 528 $H^+ \to \tau^+\nu_\tau$ events. Therefore, in all cases a higher luminosity would be necessary to observe the CP–violating asymmetry $\delta^{CP}_{\tau\nu}$.

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A. Masses and couplings of staus and sneutrinos

The mass matrix of the staus in the basis $(\tilde{\tau}_L, \tilde{\tau}_R)$,

$$\mathcal{M}^2_{\tilde{\tau}} = \begin{pmatrix} M^2_{\tilde{L}} - m_Z^2 \cos 2\beta\,((1/2) - \sin^2\theta_W) + m_\tau^2 & (A^*_\tau - \mu \tan\beta)\, m_\tau \\ (A_\tau - \mu^* \tan\beta)\, m_\tau & M^2_{\tilde{E}} - m_Z^2 \cos 2\beta \sin^2\theta_W + m_\tau^2 \end{pmatrix}. \tag{A.1}$$

It is diagonalized by a rotation matrix $R^{\tilde{\tau}}$,

$$R^{\tilde{\tau}} = \begin{pmatrix} e^{(i/2)\varphi_{\tilde{\tau}}} \cos\theta_{\tilde{\tau}} & -e^{(i/2)\varphi_{\tilde{\tau}}} \sin\theta_{\tilde{\tau}} \\ e^{-(i/2)\varphi_{\tilde{\tau}}} \sin\theta_{\tilde{\tau}} & e^{-(i/2)\varphi_{\tilde{\tau}}} \cos\theta_{\tilde{\tau}} \end{pmatrix}, \tag{A.2}$$

such that $R^{\tilde{\tau}\,\dagger}\mathcal{M}^2_{\tilde{\tau}}\, R^{\tilde{\tau}} = \mathrm{diag}(m^2_{\tilde{\tau}_1}, m^2_{\tilde{\tau}_2})$ and $\binom{\tilde{\tau}_L}{\tilde{\tau}_R} = R^{\tilde{\tau}} \binom{\tilde{\tau}_1}{\tilde{\tau}_2}$. The mass of the left-sneutrino is given by

$$m^2_{\tilde{\nu}} = M^2_{\tilde{L}} + \frac{1}{2}\, m_Z^2 \cos 2\beta\,. \tag{A.3}$$

If neutrinos have non-zero masses there is also a right-sneutrino. However, since it is electrically neutral and $h_\nu \sim 0$, it does not take part in the phenomenology discussed here. We thus neglect this state and only consider $\tilde{\nu} \equiv (\tilde{\nu})_L$.

In the following, we give the lagrangian for the interactions of (s)taus and (s)neutrinos. The other necessary parts of the interaction lagrangian are given in [2]. We start with the interaction of Higgs bosons with leptons and sleptons:

$$\begin{aligned} \mathcal{L}_{H\ell\ell} &= H^+ \bar{\nu}\, (y_\tau P_R)\, \tau^- + H^- \tau^+ (y_\tau P_L)\, \nu + H_l^0\, \tau^+ (s_l^{\tau,R} P_R + s_l^{\tau,L} P_L)\, \tau^- \,, \\ \mathcal{L}_{H\tilde{\ell}\tilde{\ell}} &= G^{\tilde{\tau}}_{4j}\, H^+\, \tilde{\nu}^*_\tau\, \tilde{\tau}_j + G^{\tilde{\tau}\,*}_{4j}\, H^-\, \tilde{\tau}^*_j\, \tilde{\nu}\,, \end{aligned} \tag{A.4}$$

with $j = 1, 2$, $l = 1, 2, 3$ and

$$P_L = \frac{1}{2}(1 - \gamma_5)\,, \qquad P_R = \frac{1}{2}(1 + \gamma_5)\,. \tag{A.5}$$

For the Higgs boson couplings to leptons we have

$$y_\tau = h_\tau \sin\beta\,, \qquad h_\tau = \frac{g\, m_\tau}{\sqrt{2}\, m_W \cos\beta}\,, \tag{A.6}$$

and

$$s_l^{\tau,R} = -\frac{g\, m_\tau}{2\, m_W}\, \left(g^S_{H_l\tau\tau} + i\, g^P_{H_l\tau\tau}\right), \tag{A.7}$$

$$s_l^{\tau,L} = -\frac{g\, m_\tau}{2\, m_W}\, \left(g^S_{H_l\tau\tau} - i\, g^P_{H_l\tau\tau}\right), \tag{A.8}$$

The $H^{\pm}$ couplings to stau and sneutrino are given by

$$G_4^{\tilde{\tau}} = \begin{pmatrix} h_\tau m_\tau \sin\beta - \sqrt{2}\, g\, m_W \sin\beta\cos\beta \\ h_\tau\,(A_\tau^* \sin\beta + \mu\cos\beta) \end{pmatrix} R^{\tilde{\tau}}\,. \tag{A.9}$$

The interactions with charginos and neutralinos are described by

$$\begin{aligned} \mathcal{L}_{\ell\tilde{\ell}\tilde{\chi}^+} &= \bar{\nu}_\tau\,(l_{ij}^{\tilde{\tau}} P_R + k_{ij}^{\tilde{\tau}} P_L)\,\tilde{\chi}_j^+\,\tilde{\tau}_i + \bar{\tau}\,(l_j^{\tilde{\nu}} P_R + k_j^{\tilde{\nu}} P_L)\,\tilde{\chi}_j^{+c}\,\tilde{\nu}_i + \\ &\quad + \overline{\tilde{\chi}_j^+}\,(l_{ij}^{\tilde{\tau}*} P_L + k_{ij}^{\tilde{\tau}*} P_R)\,\nu_\tau\,\tilde{\tau}_i^* + \overline{\tilde{\chi}_j^{+c}}\,(l_j^{\tilde{\nu}*} P_L + k_j^{\tilde{\nu}*} P_R)\,\tau\,\tilde{\nu}_\tau^*\,, \end{aligned} \tag{A.10}$$

$$\begin{aligned} \mathcal{L}_{\ell\tilde{\ell}\tilde{\chi}^0} &= \bar{\tau}\,(a_{ik}^{\tilde{\tau}} P_R + b_{ik}^{\tilde{\tau}} P_L)\,\tilde{\chi}_k^0\,\tilde{\tau}_i + \bar{\tilde{\chi}}_k^0 (a_{ik}^{\tilde{\tau}*} P_L + b_{ik}^{\tilde{\tau}*} P_R)\,\tau\,\tilde{\tau}_i^* + \\ &\quad + \bar{\nu}_\tau\,(a_k^{\tilde{\nu}} P_R + b_k^{\tilde{\nu}} P_L)\,\tilde{\chi}_k^0\,\tilde{\nu} + \bar{\tilde{\chi}}_k^0 (a_k^{\tilde{\nu}*} P_L + b_k^{\tilde{\nu}*} P_R)\,\nu_\tau\,\tilde{\nu}_\tau^*\,, \end{aligned} \tag{A.11}$$

with $i,j = 1,2$ and $k = 1,\ldots,4$. The chargino-slepton-lepton couplings are

$$\begin{aligned} l_j^{\tilde{\nu}} &= -g\,V_{j1}\,, & k_j^{\tilde{\nu}} &= h_\tau\,U_{j2}^*\,, \\ l_{ij}^{\tilde{\tau}} &= -g\,U_{j1} R_{1i}^{\tilde{\tau}} + h_\tau\,U_{j2} R_{2i}^{\tilde{\tau}}\,, & k_{ij}^{\tilde{\tau}} &= 0\,. \end{aligned} \tag{A.12}$$

The neutralino couplings to slepton and lepton are

$$\begin{aligned} a_k^{\tilde{\nu}} &= \frac{g}{\sqrt{2}}\,(\tan\theta_W N_{k1} - N_{k2})\,, & b_k^{\tilde{\nu}} &= 0\,, \\ a_{ik}^{\tilde{\tau}} &= g f_{Lk}^{\tilde{\tau}} R_{1i}^{\tilde{\tau}} + h_{Rk}^{\tilde{\tau}} R_{2i}^{\tilde{\tau}}\,, & b_{ik}^{\tilde{\tau}} &= h_{Lk}^{\tilde{\tau}} R_{1i}^{\tilde{\tau}} + g f_{Rk}^{\tilde{\tau}} R_{2i}^{\tilde{\tau}}\,, \end{aligned} \tag{A.13}$$

with

$$\begin{aligned} f_{Lk}^{\tilde{\tau}} &= \frac{1}{\sqrt{2}}\,(\tan\theta_W N_{k1} + N_{k2})\,, \\ f_{Rk}^{\tilde{\tau}} &= -\sqrt{2}\,\tan\theta_W N_{k1}^*\,, \\ h_{Lk}^{\tilde{\tau}} &= -h_\tau\,N_{k3}^* = (h_{Rk}^{\tilde{\tau}})^*\,. \end{aligned} \tag{A.14}$$

The interaction with W bosons is given by

$$\begin{aligned} \mathcal{L}_{\ell\ell W} &= -\frac{g}{\sqrt{2}}\,(W_\mu^+ \bar{\nu}_\tau\,\gamma^\mu P_L\,\tau + W_\mu^- \bar{\tau}\,\gamma^\mu P_L\,\nu_\tau)\,, \\ \mathcal{L}_{\tilde{\ell}\tilde{\ell}W} &= -i\frac{g}{\sqrt{2}}[R_{1i}^{\tilde{\tau}}\,W_\mu^+ (\tilde{\nu}_\tau^* \overleftrightarrow{\partial^\mu}\,\tilde{\tau}_i) + R_{1i}^{\tilde{\tau}*}\,W_\mu^- (\tilde{\tau}_i^* \overleftrightarrow{\partial^\mu}\,\tilde{\nu}_\tau)]\,, \end{aligned} \tag{A.15}$$

where

$$A \overleftrightarrow{\partial^\mu} B = A\,(\partial_\mu B) - (\partial_\mu A)\,B\,. \tag{A.16}$$

B. CP-violating form factors

B.1 Vertex graphs

B.1.1 Neutralino-chargino-sneutrino (stau) loop

The graph of figure $1a$, with a neutralino, a chargino, and a sneutrino in the loop, leads to

$$
\begin{aligned}
\mathrm{Re}\,\delta Y_\tau^{CP}(\tilde\chi^0_k\tilde\chi^\pm_j\tilde\nu) \;=\; & \frac{1}{8\pi^2}\Big\{\mathrm{Im}(F^L_{jk}a^{\tilde\nu}_k k^{\tilde\nu *}_j)\,\mathrm{Im}(B_0(m^2_{H^+},m^2_{\tilde\chi^0_k},m^2_{\tilde\chi^+_j}))+\\
& +\Big[m_\tau m_{\tilde\chi^0_k}\,\mathrm{Im}(F^R_{jk}a^{\tilde\nu}_k l^{\tilde\nu *}_j)+m_{\tilde\chi^+_j}m_{\tilde\chi^0_k}\,\mathrm{Im}(F^R_{jk}a^{\tilde\nu}_k k^{\tilde\nu *}_j)+\\
& +m^2_{\tilde\nu}\,\mathrm{Im}(F^L_{jk}a^{\tilde\nu}_k k^{\tilde\nu *}_j)\Big]\,\mathrm{Im}(C_0)+m_\tau\times\\
& \times\Big[m_\tau\,\mathrm{Im}(F^L_{jk}a^{\tilde\nu}_k k^{\tilde\nu *}_j)+m_{\tilde\chi^0_k}\,\mathrm{Im}(F^R_{jk}a^{\tilde\nu}_k l^{\tilde\nu *}_j)+m_{\tilde\chi^+_j}\,\mathrm{Im}(F^L_{jk}a^{\tilde\nu}_k l^{\tilde\nu *}_j)\Big]\times\\
& \times\,\mathrm{Im}(C_2)\Big\}\,,
\end{aligned}
\tag{B.1}
$$

with $C_X = C_X(0,m^2_{H^+},m^2_\tau,m^2_{\tilde\nu},m^2_{\tilde\chi^0_k},m^2_{\tilde\chi^+_j})$, $X=0,2$, the three-point functions [7] in the notation of [8]. The contribution from the neutralino-chargino-stau loop has exactly the same structure. Therefore, Re $\delta Y_\tau^{CP}(\tilde\chi^0_k\tilde\chi^\pm_j\tilde\tau_i)$ is obtained from eq. (B.1) by the following substitutions: for the masses of the loop particles $m_{\tilde\chi^0_k}\to m_{\tilde\chi^+_j}$, $m_{\tilde\chi^+_j}\to m_{\tilde\chi^0_k}$, $m_{\tilde\nu}\to m_{\tilde\tau_i}$ and for the couplings $a^{\tilde\nu}_k\to l^{\tilde\tau}_{ij}$, $b^{\tilde\nu}_k\to k^{\tilde\tau}_{ij}$, $k^{\tilde\nu *}_j\to b^{\tilde\tau *}_{ik}$, and $l^{\tilde\nu *}_j\to a^{\tilde\tau *}_{ik}$.

B.1.2 Sneutrino-stau-neutralino loop

The sneutrino-stau-neutralino loop of figure 1b gives

$$
\mathrm{Re}\;\delta Y_\tau^{CP}(\tilde\nu\,\tilde\tau_j\tilde\chi^0_k)=\frac{1}{8\pi^2}\left[m_{\tilde\chi^0_k}\;\mathrm{Im}(G^{\tilde\tau}_{4j}a^{\tilde\nu}_k b^{\tilde\tau *}_{jk})\;\mathrm{Im}(C_0)-m_\tau\;\mathrm{Im}(G^{\tilde\tau}_{4j}a^{\tilde\nu}_k a^{\tilde\tau *}_{jk})\;\mathrm{Im}(C_2)\right], \tag{B.2}
$$

with $C_X = C_X(0,m^2_{H^+},m^2_\tau,m^2_{\tilde\chi^0_k},m^2_{\tilde\nu},m^2_{\tilde\tau_j})$.

B.1.3 W boson-neutral Higgs-tau loop

For the W boson in the loop we use the $\xi=1$ gauge. We thus have to add the corresponding graph with a charged ghost, i.e. $W^\pm\to G^\pm$ in figure 1c. We get:

$$
\begin{aligned}
\mathrm{Re}\,\delta Y_\tau^{CP}(WH_l\tau) \;=\; & -\frac{\sqrt{2}g^2}{32\pi^2}\Big\{\mathrm{Im}(X^R_\tau)[(3m^2_\tau-2m^2_{H_l})\,\mathrm{Im}(C_0)+2m^2_\tau\;\mathrm{Im}(C_2)+\\
& +\mathrm{Im}(B_0(m^2_{H^+},m^2_W,m^2_{H_l}))-2\,\mathrm{Im}(B_0(0,m^2_\tau,m^2_W))]+\\
& +m^2_\tau\;\mathrm{Im}(X^L_\tau)\,\mathrm{Im}(2C_0+C_2)\Big\}\,,
\end{aligned}
\tag{B.3}
$$

$$
\mathrm{Re}\;\delta Y_\tau^{CP}(GH_l\,\tau) \;=\; -\frac{1}{8\pi^2}\,m_\tau h_\tau\cos\beta\;\left[\mathrm{Im}\,(\hat X^R_\tau)\;\mathrm{Im}\,(C_0)-\mathrm{Im}\,(\hat X^L_\tau)\;\mathrm{Im}\,(C_2)\right]\,, \tag{B.4}
$$

where $X^{R,L}_\tau = g_{H_lH^+W^-}\;s^{\tau,R,L}_l$, $\hat X^{R,L}_\tau = g_{H_lH^+G^-}\;s^{\tau\;R,L}_l$, and $C_X = C_X(0,m^2_{H^+},m^2_\tau,m^2_\tau,m^2_W,m^2_{H_l})$.

B.2 Self-energy graphs

B.2.1 Neutralino-chargino loop

The self-energy graph with a neutralino and a chargino of figure 1d gives

$$
\begin{aligned}
\mathrm{Re}\;\delta Y_\tau^{CP}\,(\tilde\chi^0_k\tilde\chi^\pm_j-W) \;=\; & \frac{1}{8\pi^2}\,\frac{g^2\,m_\tau}{\sqrt{2}\,m^2_{H^+}\,m^2_W}\;\mathrm{Im}(B_0(m^2_{H^+},m^2_{\tilde\chi^0_k},m^2_{\tilde\chi^+_j}))\times\\
& \times\Big[\mathrm{Im}\,(c_{II})\,m_{\tilde\chi^+_j}(m^2_{H^+}+m^2_{\tilde\chi^0_k}-m^2_{\tilde\chi^+_j})-\\
& -\mathrm{Im}(c_{IJ})m_{\tilde\chi^0_k}(m^2_{H^+}-m^2_{\tilde\chi^0_k}+m^2_{\tilde\chi^+_j})\Big]
\end{aligned}
\tag{B.5}
$$

with $c_{II}=F^R_{jk}O^R_{kj}+F^L_{jk}O^L_{kj}$, and $c_{IJ}=F^R_{jk}O^L_{kj}+F^L_{jk}O^R_{kj}$.

B.2.2 Sneutrino-stau and stop-sbottom loops

The graph of figure 1e with stau and sneutrino leads to

$$\begin{aligned}\mathrm{Re}\ \delta Y_\tau^{CP}\,(\tilde\nu\,\tilde\tau_j - W) \;=\; & \frac{g^2}{16\pi^2}\,\frac{m_\tau}{m_{H^+}^2 m_W^2}\,(m_{\tilde\tau_j}^2 - m_{\tilde\nu}^2)\times \\ & \times \mathrm{Im}\,(G_{4j}^{\tilde\tau} R_{1j}^{\tilde\tau *})\ \mathrm{Im}(B_0(m_{H^+}^2, m_{\tilde\tau_j}^2, m_{\tilde\nu}^2))\,. \end{aligned} \tag{B.6}$$

The analogous graph with stop and sbottom gives

$$\begin{aligned}\mathrm{Re}\ \delta Y_\tau^{CP}\,(\tilde t_i\,\tilde b_j - W) \;=\; & \frac{3g^2}{16\pi^2}\,\frac{m_\tau}{m_{H^+}^2 m_W^2}\,(m_{\tilde t_i}^2 - m_{\tilde b_j}^2)\times \\ & \times \mathrm{Im}\,(G_{4ij}^{\tilde t} R_{1i}^{\tilde t} R_{1j}^{\tilde b *})\ \mathrm{Im}\left(B_0(m_{H^+}^2, m_{\tilde b_j}^2, m_{\tilde t_i}^2)\right), \end{aligned} \tag{B.7}$$

where $G_{4ij}^{\tilde t}$ is the $\tilde t\tilde b H^+$ coupling, see [2, eqs. (48) and (49)].

B.2.3 $W^\pm$–H_l^0 and $G^\pm$–H_l^0 loops

The self-energy graph with W^+ and H_l^0 is shown in figure 1f. Since we use $\xi = 1$ gauge for the W in the loop, we have to add the corresponding graph with a ghost, i.e. $W^\pm \to G^\pm$ in the loop. (The second W propagator can be calculated in the unitary gauge. Hence, no ghost is necessary in this case.) The two contributions together give:

$$\begin{aligned}\mathrm{Re}\ \delta Y_\tau^{CP}\,(W H_l^0 - W) \;=\; & -\frac{1}{32\pi^2}\,\frac{g^3\, m_\tau}{\sqrt{2}\, m_{H^+}^2 m_W}\,(2m_W^2 - 2m_{H_l}^2 - 3\, m_{H^+}^2)\times \\ & \times O_{3l}\,(\cos\beta\, O_{1l} + \sin\beta\, O_{2l})\ \mathrm{Im}\left(B_0(m_{H^+}^2, m_{H_l}^2, m_W^2)\right). \end{aligned} \tag{B.8}$$

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